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# THE DEGENERATION OF THE HOUGH-FUNCTIONS WITHIN THE THERMOSPHERE

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THE DEGENERATION OF THE HUGH-FUNCTIONS WITHIN THE THERMOSPHERE

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ABSTRACT

Within the lower non-dissipative atmosphere, the eigenfunctions of the atmosphere are the Hough-functions which describe the tidal wave motions. Under the influence of dissipation mechanisms like heat conduction, viscosity and neutral-ion drag these Hough-functions become modified at thermospheric heights and are shown to degenerate ultimately into spherical harmonics at heights above 200 km. The implications of this degeneration on the latitudinal structure of the tidal modes and on the vertical wave propagation are discussed. It is suggested in particular that the semi-diurnal mode becomes quasitrapped at altitudes above 160 km thus explaining the apparent absence of semi-diurnal variations in the dynamics of the upper thermosphere.

A common way to treat thermosphere dynamics is to separate the variables and this is only possible under the assumption that perturbation theory is a good approximation. One asks for solutions of the form

$$F(z) \Theta(\delta) \Lambda(\lambda, t) \quad (1)$$

where  $F$  is a function of height  $z$ ,  $\Theta$  is a function of co-latitude  $\delta$  and  $\Lambda$  is a function of longitude  $\lambda$  and universal time  $t$ . In the case of regular tidal and planetary waves,  $\Lambda$  is a simple exponential function

$$\Lambda = \exp [jm\lambda + j\omega t] \quad (2)$$

where  $m$  is an integer and  $\omega$  is the angular frequency of the wave. It remains then to determine the two functions  $F$  and  $\Theta$  in (1).

Since the earth's atmosphere is a cavity, it behaves like a spherical wave guide in which only discrete geometrical structures of the waves can exist. The eigenfunctions that describe these wave modes are the Hough-functions within the lower atmosphere [1] and are identical with the function  $\Theta$  in (1) in this height region. Hough-functions can be represented by a series of spherical harmonics and generally have a rather complicated latitudinal structure due to the strong influence of the Coriolis force on the wave propagation. However, within the thermosphere, dissipation effects like heat conduction, viscosity and neutral-ion drag become increasingly important with increasing heights and reduce the influence of the Coriolis force. The Hough-functions therefore change their latitudinal structures with altitude between about 100 and 200 km and ultimately degenerate into the spherical harmonics above about 200 km [2]:

$$\Theta_n^{m,f} \rightarrow p_n^m \quad \text{above 200 km} \quad (3)$$

The indices  $n$  and  $m$  of the function  $\Theta$  are identical with the wave domain numbers  $n$  and  $m$  of the corresponding spherical function  $p_n^m$ , and

$$f = \omega/2\Omega \quad (4)$$

is the Coriolis parameter ( $\Omega$  is the angular frequency of the earth's rotation).

Figure 1 shows as an example the pressure and the horizontal wind components of the fundamental symmetric diurnal tidal wave mode  $\Theta_1^{1,1/2}$  of gravity wave type. In Figure 1a the pressure function of this mode is plotted versus co-latitude for four different heights between 100 and 300 km. At 100 km altitude a non-dissipative atmosphere has been assumed. Here the Hough-function is the trapped (1,-1)-mode of Kato [3] or the (1,-2)-mode of Lindzen [4]. The dash-dotted line of the pressure function at 100 km gives the exact structure of this mode while the solid line represents an approximation introduced to simplify the analytic solutions. The dashed lines give the phase of the (generally complex) pressure function which is zero at 100 km indicating that  $\Theta$  is a real function there.

Between 100 and 200 km, the structure of the pressure function changes until finally it is transformed into the spherical harmonic  $P_1^1$  which describes there the thermospheric diurnal pressure bulge of the Jacchia-model [5].

In Figure 1b and 1c, the meridional and zonal wind components (blowing from the north and from the west, respectively) of the diurnal mode  $\Theta_1^{1,1/2}$  are plotted versus co-latitude in the same manner as in the case of the pressure function in Figure 1a. Here the zonal wind at 100 km altitude and below changes

its direction at about  $30^\circ$  latitude which is indicated by a phase jump of 12 hours. This wind field is mainly responsible for the generation of the geomagnetic Sq current at E-layer heights [3], [6], [7].

During its transition within the height range between 100 and 200 km, this reversal of the zonal wind direction disappears, and the horizontal wind field above 200 km is essentially identical with the wind field derived by Geisler [8] and by Kohl and King [9] from the Jacchia-model [5].

The transition region between 100 and 200 km of this wave mode and likewise of all the other wave modes, too, imposes a problem in connection with our basic assumption that the variables in Eq. (1) can be separated. As we note from Figure 1, within this height range the latitude functions  $\Theta$  depend not only on co-latitude but also on height which contradicts this assumption. Apparently, a coupling between the various wave modes is to be expected in that height range. The degree of coupling can only be determined by an exact solution of the system of partial differential equations without separation of the variables. If we nevertheless do such a separation as in Figure 1 and consider the different wave modes as if they were decoupled from each other we in fact make a kind of ray optics approximation well known from the electromagnetic wave propagation within the ionosphere.

A coupling between the various wave modes takes place although perturbation approximation has been assumed. Taking into account the non-linear terms in the hydrodynamic equations, gives rise to an additional coupling between the modes and complicates the problem furthermore [10], [11].

Assuming the separation of the variables as a sufficient approximation, one obtains separation constants for the various wave modes called the equivalent depths. These equivalent depths are a measure for the eigenvalues of the height

function  $F$  in Eq. (1). Taking into account heat conduction, viscosity and neutral -ion drag, gives 8 complex eigenvalues corresponding to 8 characteristic solutions or characteristic waves of the system of the 8 complex coupled ordinary differential equations of first order for the height function  $F$  of each wave mode. Within a thin strip at a certain height in which the physical parameters of the thermosphere can be considered as constant, the characteristic waves are decoupled from each other, and each characteristic wave has the height structure

$$\exp[-(j\alpha_i + \beta_i - 1) z/2H_0] \quad (5)$$

where  $\alpha_i$  and  $\beta_i$  are real and imaginary part, respectively, of the eigenvalues of the  $i^{\text{th}}$  characteristic wave of the wave mode  $(n,m,f)$ , and  $H_0$  is the pressure scale height within this strip.

Within the lower atmosphere, only characteristic waves of gravity wave type can exist. They are either propagation modes ( $\alpha_i \neq 0$ ;  $\beta_i = 0$ ) or trapped modes ( $\alpha_i = 0$ ;  $\beta_i \neq 0$ ). The  $(1,-1)$  -mode is such a trapped mode with  $|\beta_i| > 1$ . Within the thermosphere, heat conduction waves and two kinds of viscosity waves can exist besides the gravity waves. Each wave type consists of an upgoing wave ( $\beta_i > 0$ ) and a downgoing wave ( $\beta_i < 0$ ). Above about 200 km, all characteristic waves are quasi-trapped ( $\alpha_i \neq 0$ ;  $|\beta_i| > 1$ ).

In Figure 2, we plotted versus altitude the attenuation factors  $\beta - 1$  and the propagation factors  $\alpha$  of the upgoing gravity wave type of three tidal wave modes - the fundamental symmetric diurnal mode  $\Theta_1^{1,1/2}$  or the  $(1,-1)$  -mode of Kato [3], the fundamental symmetric semi-diurnal mode  $\Theta_2^{2,1}$  or the  $(2,2)$  -mode according to Chapman and Lindzen [1] and the fundamental antisymmetric diurnal mode  $\Theta_{\frac{1}{2}}^{1,1/2}$ . The  $\Theta_1^{1,1/2}$  -mode being a trapped mode within the lower



atmosphere remains a quasi-trapped mode at thermospheric heights with decreasing attenuation factor above 140 km. The semidiurnal  $\Theta_2^{2,1}$  -mode is a propagation mode at 100 km ( $\beta = 0$ ). However, with increasing height, the attenuation factor  $\beta$  decreases rapidly (the dashed line in Figure 2a which indicates negative values of  $\beta - 1$ ) and becomes greater than one at 160 km. Above this height, the  $\Theta_2^{2,1}$  -mode behaves like a quasi-trapped mode. The antisymmetric  $\Theta_2^{1,1/2}$  -mode which describes the annual modulation of the diurnal variation of pressure and winds has the eigenvalue ( $\beta = 1$ ;  $\alpha = 0$ ) within the lower atmosphere. This mode too changes its character into a quasi-trapped mode above 100 km.

The propagation factors  $\alpha$  which are a measure for the vertical phase velocities increase with altitude between 100 and 150 km and then decrease reaching asymptotic values above about 300 km. At each height one can define a vertical wave length according to

$$\lambda_z = 4 \pi H_0 / \alpha \quad (6)$$

The upper abscissa in Figure 2b is scaled according to  $\lambda_z/H_0$  and shows that at 150 km the tidal waves have minimum vertical wave lengths.

Figure 2 shows only the one most important upgoing characteristic gravity wave for the three respective tidal modes. This characteristic wave together with the seven others can be excited in any height by a heat source component corresponding to the respective tidal mode. The eight characteristic waves propagate up and down and become coupled with each other at any height within a realistic thermosphere.

If one considers a heat input limited within a certain height region between  $z_0$  and  $z_1$ , the boundary conditions are the radiation conditions which

imply zero wave energy flux from outside through the boundaries  $(z_0, z_1)$  of the model. These conditions are fulfilled by setting the incoming characteristic waves from above (that are waves with  $\beta_i < 0$ ) equal to zero at the upper boundary  $z_1$  and by setting waves from below ( $\beta_i > 0$ ) equal to zero at the lower boundary  $z_0$ .

Such procedure can be handled without great mathematical difficulty in numerical calculations if one neglects the viscosity waves [12]. Otherwise, one must use the conditions of zero flux of mass, momentum and energy through the upper boundary as an alternative upper boundary condition [13], [14], [15] by which one loses however the advantage of the discriminating various characteristic waves.

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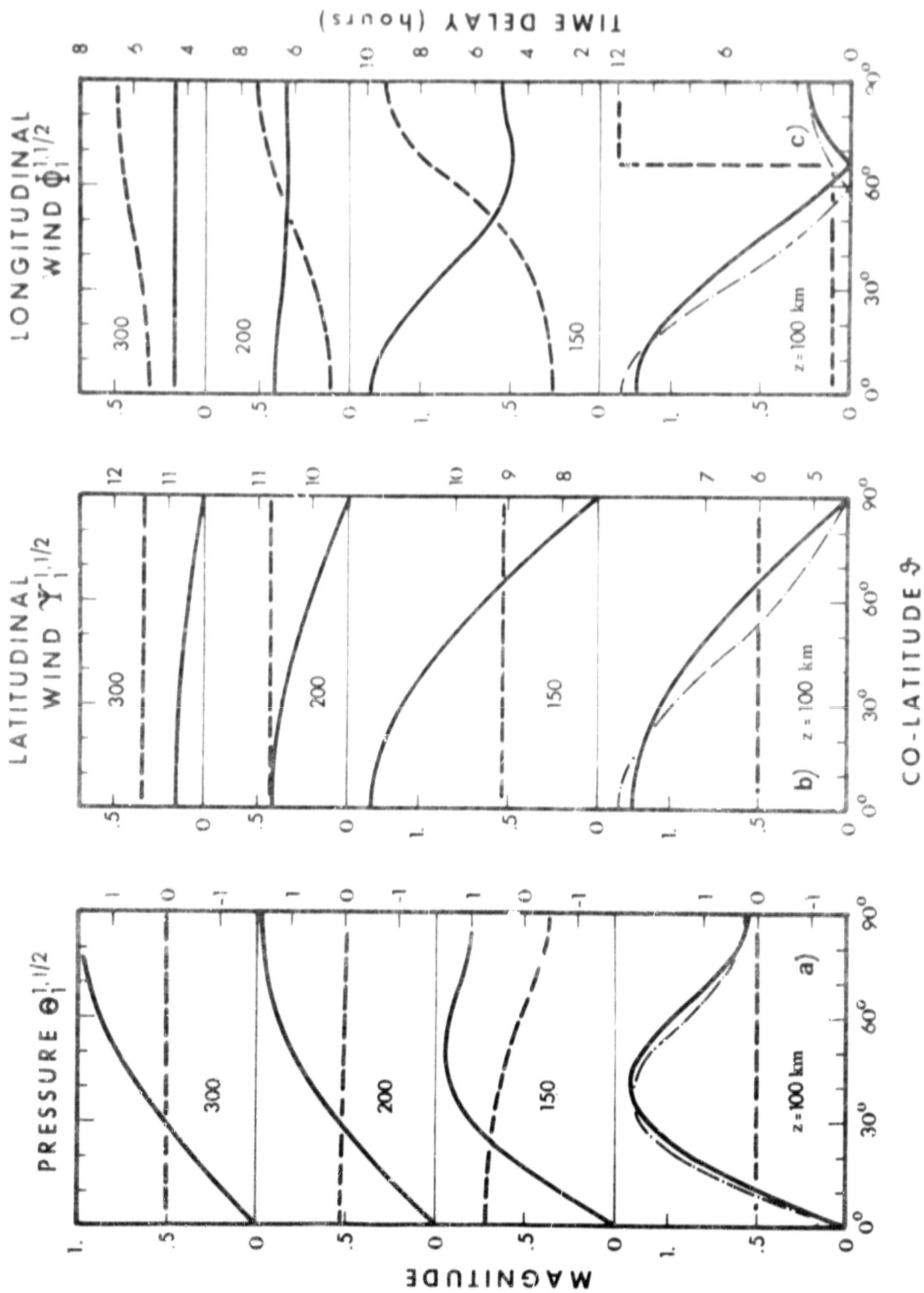


Figure 1. Dependence on co-latitude  $\vartheta$  of the fundamental symmetric diurnal tidal mode  $\Theta_1^{1,1/2}$  for four different heights. (a) Pressure function; (b) Zonal wind; (c) Meridional wind. Solid lines - Magnitudes; Dashed lines - Time delay (in hours); Dotted lines - Exact values within the non-dissipative lower atmosphere.

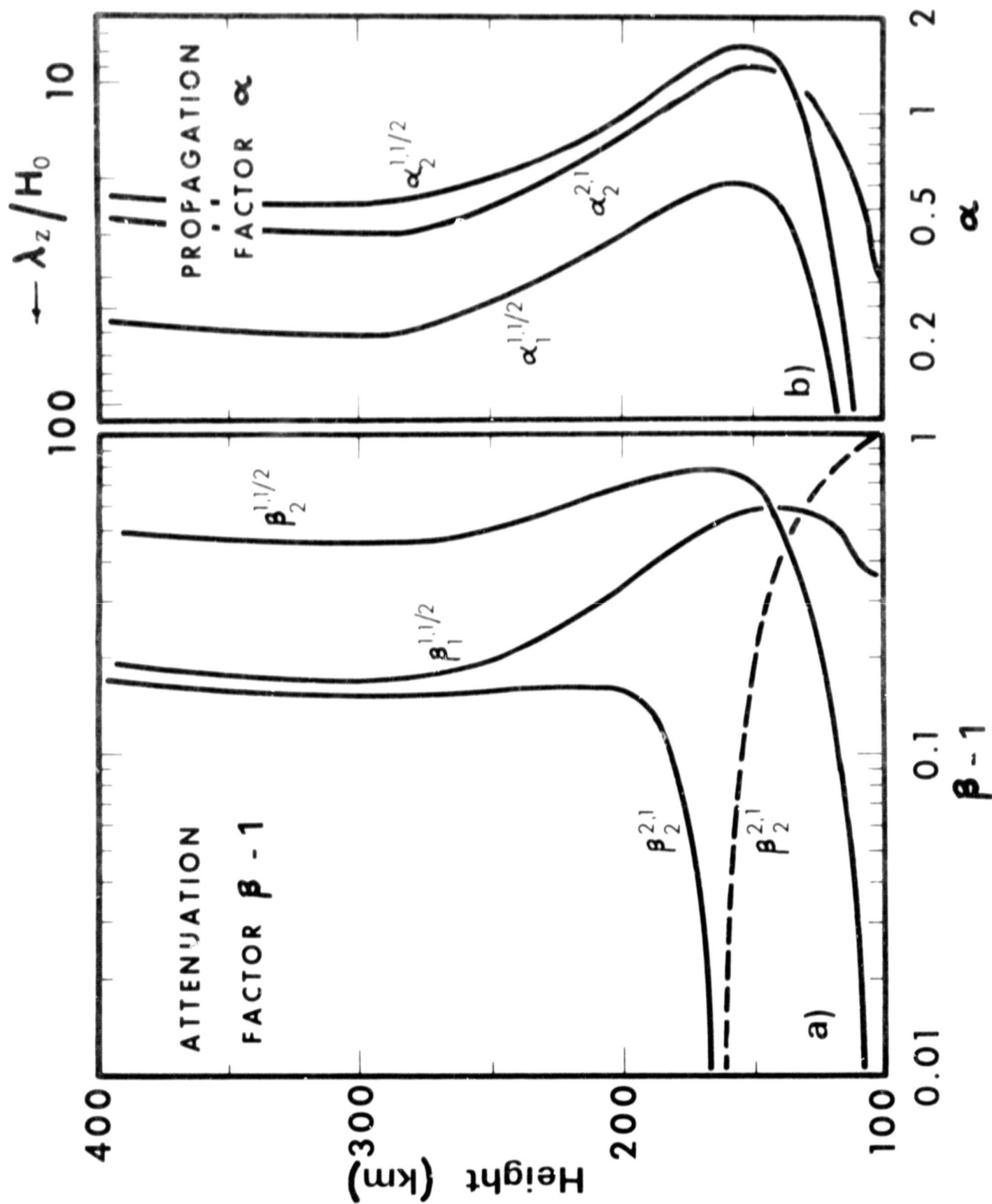


Figure 2. Normalized eigenvalues of three tidal wave modes versus altitude. (a) Attenuation factors  $\beta - 1$  ( $\beta$  is the imaginary part of the eigenvalue)  $\beta_{2,1}^{-1}$  is negative below 160 km (the dashed line). (b) Propagation factors  $\alpha$  ( $\alpha$  is the real part of the eigenvalue).